On the Diversity-Multiplexing Tradeoff of an Improved Amplify-and-forward Relaying Strategy

Yong Xiao, Luiz A. DaSilva and Xiao Juan Zhang

Abstract—**The diversity-multiplexing tradeoff (DMT) is one of the most important criteria to evaluate the performance of wireless relay networks. Previous work shows that, compared to decode-and-forward (DF) and estimate-and-forward (EF) relaying protocols, amplify-and-forward (AF) achieves the worst DMT and cannot reach the DMT upper bound. In this paper, we propose a new relaying protocol, called dynamic AF (DAF), which allows the relay to adapt the receiving and forwarding time durations to the channel conditions. We show that DAF achieves the DMT upper bound when the multiplexing gain r is between 0 and α, where $0 < \alpha < 0.5$ is a constant set according to the relaying channel conditions.**

I. Introduction

The diversity and multiplexing tradeoff (DMT) is an effective tool to characterize the performance of multi-antenna communication systems [1]–[3]. Recently, a relay-based technique called cooperative diversity has been introduced to allow multiple users to help each other to form a virtual multiple-antenna system [4]. It was shown that, under certain conditions, cooperative diversity can achieve similar performance as MIMO systems without requiring a physical multi-antenna array to be installed on each node [5]–[7].

In this paper, we consider a single-relay channel [8] in which a source transmits information to a destination with the help of a relay. Three main relaying protocols have been proposed for this channel. The first one is decode-and-forward (DF) [9], where the relay decodes all the received signals and sends the re-encoded symbols to the destination. The second one is called estimate-and-forward (EF) [10], in which the relay forwards an "estimated" version of its received signal. Amplify-and-forward (AF) [11] is the third one, in which the relay directly forwards a scaled version of its received signal to the destination. Generally speaking, AF has the lowest computational complexity among the three protocols. However, it also suffers from some performance loss compared to DF and EF. Specifically, it was proved in [5] that EF is the only protocol that can achieve the DMT upper bound of the MIMO system when the multiplexing gain r is between 0 and 1. The dynamic DF (DDF) proposed in [6] can achieve the same upper bound only when $0 \leq r < 0.5$. It was shown in [12] that the achievable DMT of orthogonal AF (OAF), in which the source remains silent when the relay forwards the signal, is much lower than the MIMO DMT upper bound. To improve the performance of OAF, the work in [6] studied the nonorthogonal AF (NAF) and proved that, by allowing the source to transmit during the entire transmission process, the DMT of OAF can be greatly improved. A new protocol, called slotted AF (SAF), was later proposed in [13], which can further improve the DMT of NAF for multiple-relay cases. Nevertheless, it was reported that even the genie-aided SAF, where the relay is assumed to know the coded source signal before transmission, cannot achieve the MIMO DMT upper bound, as EF and DDF do.

In this paper, we propose a new AF protocol, namely dynamic AF (DAF). In the proposed protocol, the relay adjusts the time durations of the receiving and forwarding operations according to the instantaneous channel conditions. The idea of improving the performance of AF by adjusting the transmitting and receiving intervals has been previously considered in [14], in which the bursty amplify-and-forward (BAF) protocol was proposed to let the transmit power of the source be concentrated and transmitted in a very short time to achieve a bursty transmission. The paper proved that if both the time fraction of the source transmission and the received signal-to-noise ratio of the destination approach zero (high source transmit power and low SNR), the optimal outage capacity of the relay channel can be achieved.

The DAF we propose in this paper is fundamentally different from BAF in the following senses: 1) In BAF, the transmit power of the source approaches infinity as the transmit interval of the source goes to zero. However, high transmit power is difficult to implement in a wireless network because of the physical power limitation of the mobile devices as well as the interference constraints of other users in the system. 2) BAF cannot achieve the DMT upper bound of DAF, DDF, and EF because it assumes the source stops transmission after the bursty transmission. DAF, as we will show, can achieve the DMT upper bound. 3) In addition, in BAF, the source needs to be coordinated with the relay to ensure the relay knows when to stop receiving signals from the source. In DAF, however, the relay will adjust its receiving and relaying time interval according to its own measurement of the channel gain, without coordination or instruction from the source.

Another result which is related to our paper is presented in [15], which extends the AF protocol to a large multi-hop network to show that AF is optimal in terms of diversity order when the SNR is large. The work in [15] assumes the multi-hop relaying operation has been pre-scheduled and focuses on the asymptotic performance of large networks. In this paper, however, we focus on a single-relay channel and characterize

Y. Xiao, L. A. DaSilva are with CTVR, Trinity College Dublin, Ireland. (email: yongx and dasilval@tcd.ie).

X. J. Zhang is with the Institute for Infocomm Research, A*STAR, Singapore. (e-mail: xzhang@i2r.a-star.edu.sg).

This material is based upon works supported by the Science Foundation Ireland under Grant No. 10/IN.1/I3007.

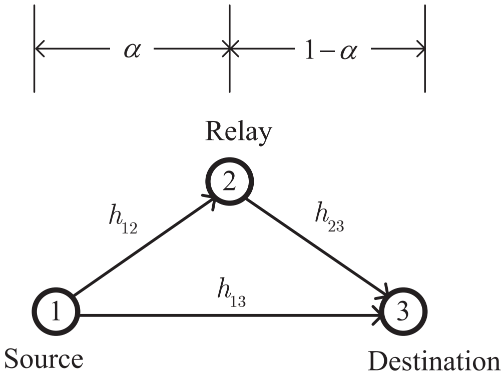

Fig. 1. Network model for the single-relay channel.

the performance of DAF in a single-relay channel. We prove that DAF can achieve the DMT upper bound if $0 \leq r \leq \alpha \leq 0.5$, where α is a constant that depends on the conditions of the source-to-relay and relay-to-destination channels. To the best of our knowledge, this is the only AF protocol that is reported to achieve the MIMO DMT upper bound.

The rest of this paper is organized as follows. The network model and background information are presented in section II. The main result and the proof are reported in sections III and V, respectively. Section IV provides discussion of the main results. This paper is concluded in section VI.

II. Network Model

Consider a single-relay system consisting of a source, a relay and a destination, labeled as 1, 2 and 3, respectively, each of which employs a single antenna, as shown in Fig. 1. The relay works in half-duplex mode and hence the entire transmission time can be divided into two frames: relay receiving frame (RR) and relay transmitting frame (RT). We consider the transmission of one codeword with length of B symbol intervals. All channels experience Rayleigh flat fading and the channel gains remain constant during each codeword.

Assume the relay listens during the first $\lfloor \alpha B \rfloor$ symbol intervals and starts to forward signals during the remaining $B - \lfloor \alpha B \rfloor$ symbol intervals. By denoting the transmitted and received signals as x and y, respectively, we can write the signals received by the destination and relay during the ith symbol interval, for $1 \leq i \leq \lfloor \alpha B \rfloor$, as

$$y_{3,i} = h_{13}x_{1,i} + z_{3,i}, \tag{1}$$

$$y_{2,i} = h_{12}x_{1,i} + z_{2,i}, \tag{2}$$

where $z_{2,i}$ and $z_{3,i}$ are zero-mean Gaussian random variables with variances σ_2 and σ_3, respectively. We assume the signals sent by the source and relay are under equal average power constraints and let the average power for source and relay be w. We also denote the signal to noise ratio at the destination as $\mathrm{SNR} = \frac{w}{\sigma_3}$.

During the RT frame, the relay randomly picks up the signal received in one symbol interval (let us call it the ith symbol interval) during the previous frame and forwards a weighed version of this signal to the destination. The signal observed by the destination during the jth symbol interval, for $\lfloor \alpha B \rfloor + 1 \leq j \leq B$, in the RT frame is given by

$$\begin{aligned} y_{3,j} &= h_{13}x_{1,j} + h_{23}x_{2,j} + z_{3,j} \\ &= h_{13}x_{1,j} + h_{23}\theta y_{2,i} + z_{3,j} \\ &= h_{13}x_{1,j} + \theta h_{23}h_{12}x_{1,i} + \theta h_{23}z_{2,i} + z_{3,j} \end{aligned} \tag{3}$$

where θ is the weighing coefficient of the relay to ensure the average power constraint is satisfied. In this paper, we consider the fixed-gain AF protocol and assume θ is a constant.

A coding strategy $\{R(\mathrm{SNR})\}$ is said to achieve spatial multiplexing gain r and diversity gain d if the data rate $R(\mathrm{SNR})$ and the average error probability $\Pr_e(\mathrm{SNR})$ satisfy the following conditions [1, Definition 1],

$$\lim_{\mathrm{SNR}\to\infty} \frac{R(\mathrm{SNR})}{\log(\mathrm{SNR})} = r, \tag{4}$$

$$\lim_{\mathrm{SNR}\to\infty} \frac{\log \Pr_e(\mathrm{SNR})}{\log(\mathrm{SNR})} = -d. \tag{5}$$

To better illustrate the performance of AF, we list the relevant previously reported results. In [5], the upper bound for the DMT of the single-relay channel achieved by EF is proven to be

$$d^{Upper}(r) = (2 - 2r)^{+}. \tag{6}$$

where $(\cdot)^{+} = \max\{\cdot, 0\}$.

This equals the DMT upper bound for a 2 by 1 MIMO system [1].

In [12], the achievable DMT for the OAF is shown to be

$$d^{OAF}(r) = 2(1 - 2r)^{+}. \tag{7}$$

In [6], it was proven that currently the best DMT result for the AF-based half-duplex single-relay channel is achieved by NAF, for which the DMT is given by

$$d^{NAF}(r) = (1 - r) + (1 - 2r)^{+}. \tag{8}$$

III. Dynamic Amplify-and-forward (DAF)

The operation of DAF is described as follows. The time durations of the RR and RT frames are denoted as $\lfloor \alpha B \rfloor$ and $B - \lfloor \alpha B \rfloor$ symbol intervals, respectively. During the RR frame, the source transmits signals to both the relay and the destination and, during the RT frame, both source and relay communicate with the destination. Differently from NAF or SAF, in DAF we assume the listening time duration of the relay depends on the instantaneous channel gains of its channel. More specifically, during the RR frame, the source transmits its information to the relay at a rate R and the relay listens until the mutual information between its received signal and the source signal exceeds BR. From the above protocol description, we have that the time fraction used by the relay to listen should be

$$\alpha \geq \min\left\{1, \frac{r\log(\mathrm{SNR})}{\log\left(1 + |h_{12}|^2 \frac{w}{\sigma_2}\right)}\right\}. \tag{9}$$

During the RT frame, the relay tries to ensure the destination can obtain exactly the same amount of information it received during the first $\lfloor \alpha B \rfloor$ symbol intervals. To achieve this, the relay needs to make sure the mutual information between its received signal and the source signal is equal to that between its forwarded signal and the signals received by the destination from the relay. We assume the relay uses the distributed amplify-and-forward method [16] to let its forwarded signal be independent from the signals received by the destination during the RR frame, and the destination only performs the decoding after it receives all the signals sent by the source and relay. That is, α needs to satisfy the following condition,

$$\alpha = \frac{M_3}{M_2 + M_3}, \tag{10}$$

where M_2 and M_3 are the channel capacity between the source and relay and between the relay and destination, i.e.,

$$\begin{aligned} M_2 &= \log\left(1 + \frac{|h_{12}|^2 w}{\sigma_2}\right), \\ M_3 &= \log\left(1 + \frac{\theta^2 |h_{12}|^2 |h_{23}|^2 w}{\sigma_3 + \theta^2 |h_{23}|^2 \sigma_2}\right). \end{aligned} \tag{11}$$

Our main result is given as follows.

Theorem 1. *DAF achieves the DMT upper bound $d^{DAF}(r) = 2 - 2r$ when $0 \leq r \leq \alpha < 0.5$, where α is given in (10).*

IV. Discussion

Figure 2 compares the DMT curves for DAF, OAF, NAF, and direct transmission. We observe that DAF is optimal when multiplexing gain r satisfies $0 \leq r < \alpha$. This is because, in this case, the relay can receive all the signals sent by the source and use equation (10) to make sure the destination can successfully obtain these signals from the relay.

However, our protocol is limited by the condition $0 \leq r \leq \alpha < 0.5$. In other words, if $r > \alpha$, the relay can only observe part of the signals sent by the source and hence DAF cannot be applied. In this case, the relay can use other AF protocols such as NAF or OAF, to forward its signals. Since α is always less than 0.5, the relay should repeatedly forward a part of or all the received signals in the RT frame. How to choose these repeated signals for DAF is a topic for our future work.

Note that, similarly to DDF, in order to decode the signal the destination needs to know the weighting coefficients of the relay, the relay listening time duration, and the channel gains between the relay and destination, between the source and destination, and between the source and relay. This can be done by allowing the source to embed a training code into its transmitted signal. In addition, the relay in DAF is also required to observe the channel gains of its channels in order to determine the value of α. This can be done by embedding training codes in the signal transmitted by the source and the destination feedback signal.

V. Proof of Theorem 1

Following the same line as [1], we assume the source 1 uses a Gaussian random code with codeword length B and the destination uses a maximum likelihood decoder to process its received signals. For data rate $R = r\log(\text{SNR})$, the error probability at the destination is given by

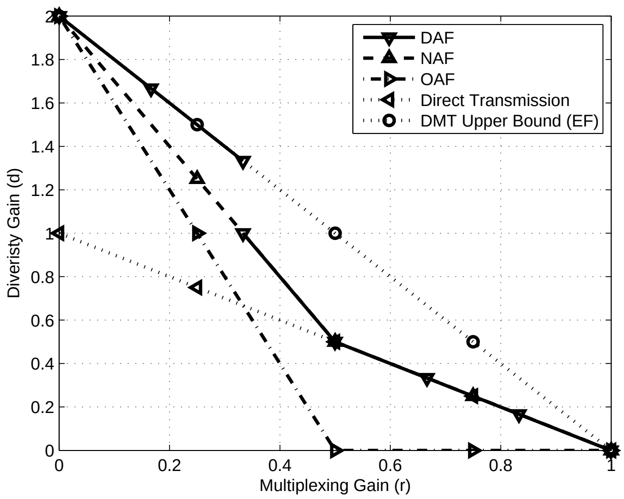

Fig. 2. DMT comparisons among different relaying protocols: we assume $\alpha = 1/3$ and, if DAF cannot be applied, it will turn to NAF.

$$\begin{aligned} \text{Pr}_e(\text{SNR}) &= \text{Pr}_o(R)\text{Pr}_{e|o}(\text{SNR}) + \text{Pr}_{e,o^c}(\text{SNR}) \\ &\leq \text{Pr}_o(R) + Pr_{e,o^c}(\text{SNR}) \end{aligned} \tag{12}$$

where e and o denote error and outage events, respectively. As proven in [1], if the codeword length B is large enough, the probability of error conditioned on the channel without outage is negligible. We then focus on the outage probability as follows. An outage occurs if the mutual information between the source and the destination is less than a rate $R = r\log(\text{SNR})$ and hence we have,

$$\begin{aligned} &\text{Pr}_o(R) = \\ &\quad \text{Pr}\left(I(X_{1,1}, ..., X_{1,B}; Y_{3,1}, ..., Y_{3,B}) \leq r\log(\text{SNR})\right) \end{aligned} \tag{13}$$

Assume the relay 2 uses the distributed amplify-and-forward method [16] to relay its received signal and hence the signals observed by the destination in RR and RT frames are independent. Thus, we have

$$\begin{aligned} &I(X_{1,1}, ..., X_{1,B}; Y_{3,1}, ..., Y_{3,B}) \\ &\quad = I(X_{1,1}, ..., X_{1,\lfloor \alpha B \rfloor}; Y_{3,1}, ..., Y_{3,\lfloor \alpha B \rfloor}) \\ &\qquad + I(X_{1,\lfloor \alpha B \rfloor+1}, ..., X_{1,B}, X_{2,\lfloor \alpha B \rfloor+1}, ..., X_{2,B}; \\ &\qquad\qquad Y_{3,\lfloor \alpha B \rfloor+1}, ..., Y_{3,B}) \\ &\quad = (\lfloor \alpha B \rfloor)\log\left(1 + \frac{|h_{13}|^2 w}{\sigma_3}\right) \\ &\qquad + (B - \lfloor \alpha B \rfloor)\log\left(1 + \frac{|h_{13}|^2 w + \theta |h_{12}|^2 |h_{23}|^2 w}{\sigma_3 + |h_{23}|^2 \sigma_2}\right) \end{aligned} \tag{14}$$

Defining v_{12}, v_{23} and v_{13} as the exponential orders of $\frac{1}{|h_{12}|^2}, \frac{1}{|h_{23}|^2}$ and $\frac{1}{|h_{13}|^2}$, respectively, and substituting (14) into (13), we have

$$\text{Pr}_o(R) \doteq \text{SNR}^{d^{DAF}(r)} \tag{15}$$

where

$$d^{DAF}(r) = \inf_{v_{12}, v_{13}, v_{23} \in O^+} v_{13} + v_{12} + v_{23}, \tag{16}$$

and

$$O^{+} = \Big\{(v_{12}, v_{23}, v_{13}) \in \mathbb{R}^{3+} | \alpha (1 - v_{13})^{+} + (1-\alpha)\left[1 - \min\{(v_{12}+v_{23}), v_{13}\}\right]^{+} \le r\Big\}. \quad (17)$$

From (10), we have

$$\alpha(1-v_{12})^{+} = (1-\alpha)\left[1 - (v_{12}+v_{23})\right]^{+} \Rightarrow v_{12} + v_{23} = 1 - \frac{\alpha(1-v_{12})}{1-\alpha}. \quad (18)$$

Substituting (18) into (16), we have

$$d^{DAF}(r) = \inf_{v_{12}, v_{23}, v_{13} \in O^{+}} v_{13} + \frac{\alpha}{1-\alpha} v_{12} + \frac{1-2\alpha}{1-\alpha} \quad (19)$$

where

$$O^{+} = \Big\{(v_{12}, v_{23}, v_{13}) \in \mathbb{R}^{3+} | \alpha (1 - v_{13})^{+} + \max\left\{\alpha(1-v_{12})^{+}, (1-\alpha)(1-v_{13})^{+}\right\} \le r\Big\}. \quad (20)$$

It can be easily shown that if either $v_{12} > 1$ or $v_{13} > 1$, the resulting diversity order will be higher than the theoretical MIMO upper bound defined in [1]. Hence, we only consider the cases in which both v_{12} and v_{13} are between 0 and 1. Let us consider two cases as follows.

1) If $\alpha(1-v_{12})^{+} \ge (1-\alpha)(1-v_{13})^{+}$, we have $\alpha(1-v_{13})^{+} + \alpha(1-v_{12})^{+} \le r$. By combining these two inequalities, we can calculate the feasible regions of v_{12} and v_{13} to be $1 - \frac{1-\alpha}{\alpha} r \le v_{12} < 1$ and $1 - r \le v_{13} < 1$. Substituting the lowest values of v_{12} and v_{13} into (19), we have $d^{DAF}(r) = 2 - 2r$,
2) If $(1-\alpha)(1-v_{13})^{+} \ge \alpha(1-v_{12})^{+}$, we also have $v_{13} > 1 - r$. Using similar methods as case 1), we have $v_{12} \ge 1 - \frac{1-\alpha}{\alpha} r$. Hence, we obtain $d^{DAF}(r) = 2 - 2r$.

Note that from (10), we have $0.5 > \alpha \ge r$. In other words, the diversity order $d^{DAF}(r) = 2 - 2r$ can only be achieved if $0.5 > \alpha \ge r \ge 0$. This concludes the proof.

VI. Conclusion

In this letter, we have introduced a new AF relaying protocol, called dynamic AF (DAF). In DAF, the relay adjusts the durations of the receiving and forwarding times according to the relay channel conditions. We prove that DAF can achieve the DMT upper bound if $0 \le r \le \alpha < 0.5$.

References

[1] L. Zheng and D. Tse, "Diversity and multiplexing: a fundamental tradeoff in multiple-antenna channels," *IEEE Trans. Inform. Theory*, vol. 49, no. 5, pp. 1073–1096, 2003.

[2] D. Tse, P. Viswanath, and L. Zheng, "Diversity-multiplexing tradeoff in multiple-access channels," *IEEE Trans. Inform. Theory*, vol. 50, no. 9, pp. 1859–1874, 2004.

[3] X. J. Zhang, Y. Gong, and K. Letaief, "On the diversity gain in MIMO channels with joint rate and power control based on noisy CSITR," *IEEE Trans. Wireless Commun.*, vol. 10, no. 1, pp. 68–72, 2011.

[4] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperation diversity — Part I: System description," *IEEE Trans. Commun.*, vol. 51, no. 11, pp. 1927–1938, 2003.

[5] M. Yuksel and E. Erkip, "Multiple-antenna cooperative wireless systems: A diversity-multiplexing tradeoff perspective," *IEEE Trans. Inform. Theory*, vol. 53, no. 10, pp. 3371–3393, 2007.

[6] K. Azarian, H. El Gamal, and P. Schniter, "On the achievable diversity-multiplexing tradeoff in half-duplex cooperative channels," *IEEE Trans. Inform. Theory*, vol. 51, no. 12, pp. 4152–4172, 2005.

[7] X. J. Zhang, Y. Gong, and K. Letaief, "On the diversity gain in cooperative relaying channels with imperfect CSIT," *IEEE Trans. Commun.*, vol. 58, no. 4, pp. 1273–1279, 2010.

[8] T. Cover and J. Thomas, *Elements of information theory*. New York: Wiley, 1999.

[9] E. Van Der Meulen, "Three-terminal communication channels," *Advances in Applied Probability*, vol. 3, pp. 120–154, 1971.

[10] T. Cover and A. Gamal, "Capacity theorems for the relay channel," *IEEE Trans. Inform. Theory*, vol. 25, no. 5, pp. 572–584, 1979.

[11] G. Kramer, M. Gastpar, and P. Gupta, "Cooperative strategies and capacity theorems for relay networks," *IEEE Trans. Inform. Theory*, vol. 51, no. 9, pp. 3037–3063, 2005.

[12] J. N. Laneman, D. Tse, and G. W. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Trans. Inform. Theory*, vol. 50, no. 12, pp. 3062–3080, 2004.

[13] S. Yang and J. Belfiore, "Towards the optimal amplify-and-forward cooperative diversity scheme," *IEEE Trans. Inform. Theory*, vol. 53, no. 9, pp. 3114–3126, 2007.

[14] A. Avestimehr and D. Tse, "Outage capacity of the fading relay channel in the low-SNR regime," *IEEE Trans. Inform. Theory*, vol. 53, no. 4, pp. 1401–1415, 2007.

[15] S. Borade, L. Zheng, and R. Gallager, "Amplify-and-forward in wireless relay networks: Rate, diversity, and network size," *IEEE Trans. Inform. Theory*, vol. 53, no. 10, pp. 3302–3318, 2007.

[16] J. Yindi and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. Wireless Commun.*, vol. 5, no. 12, pp. 3524–3536, 2006.