A Simple Distributed Power Control Algorithm for Cognitive Radio Networks

Yong Xiao, Guoan Bi, *Senior Member, IEEE,* and Dusit Niyato, *Member, IEEE*

***Abstract*—This paper studies the power control problem for spectrum sharing based cognitive radio (CR) networks with multiple secondary source-to-destination (SD) pairs. A simple distributed algorithm is proposed for the secondary users (SUs) to iteratively adjust their transmit powers to improve the performance of the network. The proposed algorithm does not require each SU (or PU) to negotiate with other SUs (or PUs) during the communication. It is proved that the proposed algorithm can obtain a time average performance as good as that achieved when the Nash equilibrium (NE) is chosen in hindsight. More specifically, the average performance of CR networks will converge to an ϵ-Nash equilibrium at a rate of $T_\epsilon = O\left(\exp\left(\frac{1}{\epsilon}\right)\right)$. A sub-optimal algorithm is also introduced to further improve the convergence rate to $\frac{T_{\epsilon'}}{\log T_{\epsilon'}} = O\left(\frac{1}{\epsilon'}\right)$. Numerical results are presented to show the performance of the proposed algorithm under different settings.**

I. Introduction

Cognitive radio (CR) is one of the main technologies to solve the spectrum under-utilization problem in future generations of wireless systems. In CR networks, the unlicensed users, called secondary users (SU), can access the spectrum which is unoccupied or ineffectively used by the licensed users, called primary users (PU). In this paper, we focus on spatial spectrum sharing (SSS) based CR networks (spectrum underlay approach) [1] in which SUs and PUs can transmit signals at the same time over the same spectrum. In this system, maximizing the performance of SUs and simultaneously maintaining the interference powers of PUs under the acceptable levels, called the interference temperature limit [1], is still a challenging task.

In [2], it was observed that, if SUs employ the power control methods, i.e., to decrease (or increase) their transmit powers when the conditions of SU-to-PU channels are "good" (or "bad"), the spectrum utilization efficiency and system performance can be greatly improved. However, most of the previously reported work neglects the interference among SUs or PUs and assumes each SU can have the global information, i.e., the states of other SUs and the channel gains of all channels in the network. This requires each SU to handle high complexity computation and unrealistic channel estimation, which may impossible for many practical systems. More specifically, in [2] [3], the optimal power control methods for the case of one secondary source-to-destination (SD) pair sharing the spectrum with one PU were derived by assuming the transmitters to know the instantaneous channel gains of both SU-to-SU and SU-to-PU channels. The results were extended to CR networks with multiple SD pairs [4] and multi-hop relaying SUs [5] based on the similar assumptions.

Motivated by these limitations, in this paper, a game theoretic framework is established to solve the power control problem of distributed CR networks with multiple secondary SD pairs and PUs. A simple distributed algorithm is proposed to achieve the following advantages: 1) the operations of each SU are simple and fully distributed, i.e., both SUs and PUs do not know the global information, and there is no central controller to manage the network. In addition, in our setting, each PU broadcasts the same information to all SUs and each SU cannot communicate with others or obtain specific instructions from PUs, 2) the computational complexity of each SU is low and does not depend on the number of SUs, 3) the proposed algorithm can be directly applied to other resource control problems, e.g., spectrum allocation, time scheduling, etc.

It is proved that, the CR network with this simple algorithm can achieve a time average performance as good as that achieved when the Nash equilibrium (NE) is chosen in hindsight. More specifically, we show that by using the proposed algorithm the average performance of each SU converges to an ϵ-NE at a rate of $T_\epsilon = O\left(\exp\left(\frac{1}{\epsilon}\right)\right)$. This is a surprising result because finding the NE for multi-user networks is generally a challenging task even when the number of users is small [6, Chapter 2], and most of the previously reported NE-approaching algorithms only focus on the instantaneous performance and require more stringent conditions than ours. To further improve the convergence rate, a sub-optimal algorithm is also proposed to approach a neighborhood of the NE at a rate of $\frac{T_{\epsilon'}}{\log T_{\epsilon'}} = O\left(\frac{1}{\epsilon'}\right)$. We discuss some potential extensions of our work and present numerical results to verify the performance of our algorithms under different applications.

The rest of this paper is organized as follows. The network model and problem setup are discussed in section II. The proposed algorithm and its convergence result are presented in Sections III. Applications and numerical results are given in Section IV and the paper is concluded in Section V.

II. Network Model and Game Formulation

Consider a CR network in which K secondary SD pairs, labeled as S_1 to D_1, S_2 to D_2, ..., S_K to D_K, simultaneously transmit signals in the same spectrum as M PUs, labeled as P_1, P_2, ..., P_M as shown in Figure 1. Let the channel gain between S_i and D_j be h_{S_i,D_j} for $i, j \in \{1, 2, ..., K\}$, and that between S_i and P_k be h_{S_i,P_k} for $i \in \{1, 2, ..., K\}, k \in \{1, 2, ..., M\}$.

Y. Xiao and G. Bi are with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore (email: xiao0021@e.ntu.edu.sg, egbi@ntu.edu.sg).

D. Niyato is with the School of Computer Engineering, Nanyang Technological University, Singapore (email: dniyato@ntu.edu.sg).

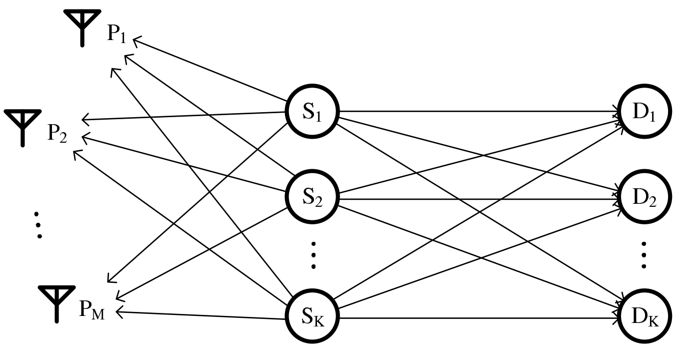

Fig. 1. Network model for CR networks with multiple SD pairs.

In a practical system, P_k for $k \in \{1, 2, ..., M\}$ can only maintain a certain level of QoS if its received interference power is lower than the interference temperature limit, denoted by q_k. By denoting the transmit power of S_i as w_i, we can define the power constraints for SUs as follows:

$$\boldsymbol{H}_{PU}\boldsymbol{w}^{\dagger} \leq \boldsymbol{q}^{\dagger}, \tag{1}$$

where $\dagger$ denotes the transpose of a matrix, $\boldsymbol{w} = [w_1, w_2, \ldots, w_K]$, $\boldsymbol{q} = [q_1, q_2, \ldots, q_M]$, and $\boldsymbol{H}_{PU} \in \mathbb{R}^{M \times K}$ is given by

$$\boldsymbol{H}_{PU} = \begin{bmatrix} h_{S_1,P_1} & h_{S_2,P_1} & \cdots & h_{S_K,P_1} \\ h_{S_1,P_2} & h_{S_2,P_2} & \cdots & h_{S_K,P_2} \\ \vdots & \vdots & \ddots & \vdots \\ h_{S_1,P_M} & h_{S_2,P_M} & \cdots & h_{S_K,P_M} \end{bmatrix}. \tag{2}$$

Based on the above notations, the received signal to noise ratio (SNR) of D_i can be written as

$$SNR_i = \frac{h_{S_i,D_i}w_i}{\sum\limits_{\substack{j\in\{1,2,\ldots,K\}\\ j\neq i}} h_{S_j,D_i}w_j + \sigma_i}. \tag{3}$$

where if PUs keep sending signals during the communication, σ_i should contain both the transmit powers of PUs and the additive noise received by S_i. However, if SUs can only transmit when PUs are absent, i.e., in the time sharing spectrum (TSS) based CR networks [7], σ_i should only contain the additive noise of S_i.

In this paper, the game theoretic method is used to investigate the power control problem for CR networks. In our game, *players* are SUs who share the spectrum with PUs. We define the *revenue* of the ith SD pair, denoted by r_i, to be the benefit obtained by S_i and D_i from using the PUs' spectrum. The *price* paid by S_i, denoted by c_i, is defined as the price charged by PUs for using the licensed spectrum. We also define the *payoff* of the ith SD pair, denoted by π_i, to be the difference between the revenue and price, i.e., $\pi_i = r_i - c_i$. In this paper, we assume the revenue and payoff of S_i are functions of SNR_i defined in (3).

In this paper, a finitely repeated game model [8] is applied in which each SU plays the game repeatedly. By using subscript $[t]$ to denote the setting and operations in the tth period, the main objective of the ith SD pair is to maximize its time average payoff $\frac{1}{T}\sum_{t=1}^{T}\pi_i(\boldsymbol{w}_{[t]})$ over T periods of transmission without causing the adverse effects on other SD pairs. This is different from most of the reported work in repeated game where each player only cares about the payoff of the last iteration, i.e., payoff in the Tth time slot. Our setting has more practical meaning for CR networks because most mobile devices can tolerate a few periods of "bad" performance if the average payoff is good.

It is assumed that each player cares the performance of the future as same as that of the present and hence the time discount factor [8, Definition 6.1.2] is 1. The results with other choice of the time discount factor can be similarly obtained. The main objective is to find a balance point of the entire network, called Nash Equilibrium (NE), in which each SD pair cannot further improve its payoff by choosing a different transmit power, given the transmit powers of other SUs. In this paper, a distributed algorithm is introduced to enable each SU to iteratively adjust its power to reach the NE. The definition of NE is given as follows.

Definition 1. *[8, Definition 3.3.4] A strategy profile w_i^* is at a Nash equilibrium (NE) if, for every player i and every strategy w_i, w_i^* is at least as good as the strategy profile (w_i, w_{-i}^*) in which the player i chooses w_i while other players choose w_{-i}^*, i.e., for every player i, $\pi_i(w_i^*, w_{-i}^*) \geq \pi_i(w_i, w_{-i}^*)$, where subscript $-i$ means all the players except player i. We also define a strategy profile to be ϵ-Nash equilibrium (ϵ-NE), if this strategy profile is within the distance of ϵ to the payoff achieved by a NE, i.e., the following condition needs to be satisfied, $\pi_i\left(w_i^*, w_{-i}^*\right) - \pi_i\left(w_i, w_{-i}^*\right) \leq \epsilon$ for $\epsilon > 0$.*

Finding the NE of the multi-user network is difficult and intractable [6, Chapter 2]. In this paper, we try to develop an algorithm that, during T periods of repeated power control games, can achieve the average performance nearly as good as the system with the NE decision being chosen in hindsight, i.e., we have

$$\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\min_{l\in\{1,\ldots,t\}}\left\|\boldsymbol{\pi}\left(\boldsymbol{w}_{[l]}\right) - \boldsymbol{\pi}\left(\boldsymbol{w}^*\right)\right\|_2^2 = 0 \tag{4}$$

where $\boldsymbol{\pi}(\boldsymbol{w}_{[l]}) = [\pi_1(\boldsymbol{w}_{[l]}), \pi_2(\boldsymbol{w}_{[l]}), ..., \pi_K(\boldsymbol{w}_{[l]})]$.

III. Main Results

Algorithm 1 is illustrated below.

Algorithm 1

1) *Initialization*: Let the transmit signals of S_i be x_i. Consider the transmission of the ith SD pair. During the first 2 time slots, i.e., $t \in \{1, 2\}$,
   a) S_i first sends signals $x_{i[1]}$and $x_{i[2]}$ with powers $w_{i[1]}$ and $w_{i[2]}$, respectively,
   b) After receiving the signals sent by S_i in the first and second time slots, D_i feedbacks the revenue functions $r_{i[1]}$ and $r_{i[2]}$, respectively, decided by its received SNR, and PUs feedback prices $c_{i[1]}$ and $c_{i[2]}$, respectively, to S_i.
2) *Iteration*: For $t = 3 : T$,
   a) In time slot t, P_k monitors its interference power $\sum_{i=1}^{K} h_{S_i,P_k}w_{i[t]}$. If the interference powers of all

PUs satisfy (1), P_k only sends price $c_{i[t]}$ to S_i, and then S_i updates its transmit power as follows,

$$w_{i[t]} = \left(w_{i[t-1]} - \delta_{i[t-1]} g_{i[t]}\right)^+, \quad (5)$$

where $(\cdot)^+ = \max\{0, \cdot\}$, $g_{i[t]}$ is a subgradient [9] of $\pi_{i[t]}$ which is a function that satisfies the relation

$$\begin{aligned}\pi_{i[t]}\left(\boldsymbol{w}_{[t]}\right) \quad &\geq \quad \pi_{i[t-1]}\left(\boldsymbol{w}_{[t-1]}\right) \\ &+ g_{i[t]}\left(\boldsymbol{w}_{[t]} - \boldsymbol{w}_{i[t-1]}\right), \quad (6)\end{aligned}$$

and $\delta_{i[t-1]}$ is the step size of the tth iteration, defined as $\delta_{i[t-1]} = \frac{u_i}{t-1}$ where u_i is the step size coefficient which is a constant controlling the iteration speed,

b) However, if the interference powers of some PUs exceed the interference temperature limit, each of these PUs should transmit more information to help SUs to adjust the transmit powers during the following iterations. More specifically, assuming P_k observes high interference power, i.e., $\sum_{i=1}^{K} h_{S_i,P_k} w_{i[t]} > q_k$, the operations of P_k, except for sending the price to SUs, are described as follows,

i) P_k broadcasts a high-interference-message to inform all SUs that its interference power exceeds the limit. S_i can decode this message and exploit it to obtain h_{S_i,P_k},

ii) Because we assume each PU monitors its interference power in each iteration, P_k can calculate the exceeding interference power value

$$I_{k[t]} = \sum_{i=1}^{K} h_{S_i,P_k} w_{i[t]} - q_k, \quad (7)$$

and the interference power change value in time slot t

$$J_{k[t]} = \sum_{i=1}^{K} h_{S_i,P_k} w_{i[t]} - \sum_{i=1}^{K} h_{S_i,P_k} w_{i[t-1]}, \quad (8)$$

iii) By assuming the channel gains can be known by the receivers (i.e., by using a training code involved in the transmit signals of SUs), P_k can exploit its received signal to calculate a constant $\Phi_{k[t]} = \sum_{i=1}^{K} h_{S_i,P_k}^2$ and broadcast this constant to all SUs,

iv) S_i, knowing h_{S_i,P_k} (as discussed in Step i)), updates its transmit power by

$$\begin{aligned}w_{i[t]} \quad &= \quad \left[w_{i[t-1]} - \delta_{i[t-1]} g_{i[t]}\right. \quad (9) \\ &\left. - h_{S_i,P_k} \Phi_{k[t-1]}^{-1}\left(I_{k[t-1]} - J_{k[t-1]}\right)\right]^+.\end{aligned}$$

If more than one PU detects higher-than-tolerate interference power, they should sequentially repeat the operations illustrated from Step i) to iv) until the transmit powers of all SUs satisfy (1).

3) *Termination*: The above process continues until the obtained payoff is close to the optimal value within an acceptable range, i.e., $\|\pi_{i[t]}(\boldsymbol{w}_{[t]}) - \pi_{i[t]}(\boldsymbol{w}^*)\|_2^2 \leq \epsilon$, or the number of time slots reaches T, i.e., $t = T$.

The main idea of Algorithm 1 is that, if the transmit powers of SUs satisfy (1), each SU maximizes its payoff by using the subgradient method, as shown in Step 2-a). If some PUs detect a higher-than-tolerable interference power, each of them broadcasts several constants, i.e., P_k broadcasts Ψ_k, I_k and J_k defined in Step 2) in Algorithm 1 to all SUs, and SUs use these constants to project their transmit powers into the convex hull defined in (1). However, simply projecting the transmit powers of all SUs to (1) requires each PU to know the information of other PUs, which does not meet the requirements that PUs cannot communicate with each other. Hence, in Algorithm 1, if more than one PU detect higher-than-tolerable interference power, each of them informs SUs to project their powers to the linear equations in the convex hull defined in (1) one by one. This setting will not result in much adverse effects on the convergence performance of Algorithm 1 because we always assume $T \gg K$ and if the transmit powers of SUs increase gradually, only a few PUs can first detect high-than-tolerable interference powers in each time slot. Therefore, the number of iterations used on the projection operations is negligible as the total number of iterations becomes large.

Note that our algorithm is different from that proposed in [10] in which each SU needs to communicate with its nearby SUs to make decisions based on the "consensus" among them. In our algorithm, each PU (or SU) cannot communicate with its nearby PUs (or SUs), and hence can be directly applied into many practical systems, i.e., the cellular based mobile network.

Theorem 1. *If the following three assumptions*

A1) *The iteration step sizes are bounded, i.e.,* $\|g_{i[t]}\|_2^2 \leq g_i^+$*, where* g_i^+ *is a constant, and*

A2) *The power changes between two iterations are bounded, i.e.,* $\|w_{i[t]} - w_{i[s]}\|_2^2 \leq w_i^+$ *for* $t \neq s$ *and* $t, s \in \{1, ..., T\}$*, where* w_i^+ *is a constant, and*

A3) *The payoff for the* i*th SD pair is a continuous function of* w_i*,*

are satisfied, then Algorithm 1 has the following properties.

P1) *It converges to a stationary point if* T *is large enough,*

P2) *If all SD pairs use Algorithm 1, the time average payoff of* S_i *converges to an* ϵ*-NE at a rate of* $O\left(\exp\left(\frac{1}{\epsilon}\right)\right)$ [1]*, i.e.,*

$$\left\|\frac{1}{T_\epsilon}\sum_{t=1}^{T_\epsilon} \boldsymbol{\pi}\left(\boldsymbol{w}_{[t]}\right) - \boldsymbol{\pi}\left(\boldsymbol{w}^*\right)\right\|_2^2 \leq \epsilon. \quad (10)$$

Proof: See Appendix A. ■

Note that the convergence performance of Algorithm 1 is closely related to the combination of two important parameters: the initial value $w_{i[1]}$ of transmit powers and the iteration step size u_i. If $w_{i[1]}$ and u_i satisfies $|w_{i[1]} - w_i^*| \approx u_i$, Algorithm 1 could approach to the neighborhood of a NE within a few iterations. However, if $w_{i[1]}$ and u_i are improperly chosen, the convergence rate of Algorithm 1 will be very slow. This slow convergence rate is due to the fact that we require w_i to reach w_i^* exactly without assuming any information about

[1] In this paper, we follows Bachmann-Landau notations: $f = O(g)$ if $\lim_{n\to\infty} \frac{f(n)}{g(n)} < +\infty$.

the systems or channel gains is known by SUs. The slow convergence performance of Algorithm 1 can be improved by assuming more information to be observed by SUs. For example, if we assume that S_i knows the payoff functions, or both the revenue and pricing functions, the optimization problem can be converted into the Lagrangian dual problem [11] which will be discussed in the next section.

Another way to improve the convergence rate of Algorithm 1 is to decrease the required accuracy of the results. In practical digital systems, mobile devices cannot adjust their parameters with an infinite accuracy, i.e., the value of w_i can only be chosen from a finite discrete set. In the rest of this section, let us consider a simple case in which the transmit power of each SU is linearly quantized into L levels and w_i can only be chosen from this finite set, i.e., $w_i \in \mathcal{W}_i$ where $\mathcal{W}_i = \{0, \frac{w_i^+}{L}, \frac{2w_i^+}{L}, ..., w_i^+\}$. Let us describe Algorithm 2 as follows.

Algorithm 2: Each SU adjusts its transmit power by using the exact same procedures as Algorithm 1. The only difference is that, in the tth iteration, the transmit power of S_i is $w_{i[t]} = l\hat{\delta}_i$ if $l\hat{\delta}_i - \frac{\hat{\delta}_i}{2} < w_{i[t]} < l\hat{\delta}_i + \frac{\hat{\delta}_i}{2}$ where $l \in \{1, 2, ..., L\}$, $\hat{\delta}_i = \frac{w_i^+}{L}$ and $w_{i[t]}$ is calculated by using equation (9) in Algorithm 1.

The NE is a balance point where each player achieves locally optimal solution and hence has no incentive to deviate from this point. In other words, we can always find a *neighborhood*, denoted as $\boldsymbol{\chi} = [\chi_1, \chi_2, ..., \chi_K]$, for the NE that all the elements in this neighborhood cannot achieve higher performance than that of the NE, i.e., $\pi_i(w_i^*, w_{-i}^*) \geq \pi_i(\boldsymbol{w}^* \pm \Delta\boldsymbol{w})$ where $\Delta\boldsymbol{w} = [\Delta w_1, \Delta w_2, ..., \Delta w_K,]$ and $0 < \Delta w_i < \chi_i$ for $i \in \{1, 2, ..., K\}$. It is easy to observe that if $\boldsymbol{w}_{[t]} - \boldsymbol{w}_{[t+1]} > \boldsymbol{\chi}$, Algorithm 2 cannot converge to a NE. Let us summarize our observations and present the main result for Algorithm 2 as follows.

Theorem 2. *If* $w_i \in \mathcal{W}_i$, $\chi_i \gg \frac{\hat{\delta}_i g_i^+}{2}$ $\forall\ i \in \{1, 2, ..., K\}$ *and Assumptions A1)-A3) in Theorem 1 are satisfied, Algorithm 2 approaches a* $\left[\frac{\hat{\delta}_1 g_1^+}{2}, \frac{\hat{\delta}_2 g_2^+}{2}, ..., \frac{\hat{\delta}_K g_K^+}{2}\right]$ *neighborhood of the NE, i.e.,* $\left\| \frac{1}{T_{\epsilon'}} \sum_{t=1}^{T_{\epsilon'}} \boldsymbol{\pi}\left(w_{i[t]}, w_{-i[t]}^* \pm \frac{\hat{\delta}_i g_i^+}{2}\right) - \boldsymbol{\pi}\left(\boldsymbol{w}^* \pm \frac{\hat{\delta}_i g_i^+}{2}\right) \right\|_2^2 \leq \epsilon'$, *at a rate of* $\frac{T_{\epsilon'}}{\log T_{\epsilon'}} = O\left(\frac{1}{\epsilon'}\right)$, *where* $\epsilon' = |\epsilon - \frac{\hat{\delta}_i g_i^+}{2}|$ *and* $\hat{\delta}_i = \frac{w_i^+}{L}$.

Proof: See Appendix B. ■

IV. Applications and Numerical Results

In this section, the proposed algorithms are applied to the network utility maximization problem to evaluate its performance. We assume that each SU has an objective to maximize its achievable rate and define the revenue function of S_i as $r_i(w_i) = \gamma_i \log(1 + SNR_i)$ where SNR_i is defined in (3) and γ_i is a constant. Since Algorithm 2 can be regarded as a special case of Algorithm 1, we only present the numerical results for Algorithm 1.

Consider a special case where SUs are far from each other and assume $SNR_i \approx \frac{h_{S_i,D_i} w_i}{\sigma_i}$. Assume PUs do not charge any prices for SUs if the transmit powers of SUs satisfy (1). It is observed that, in this case, the payoff function $\pi_i(\boldsymbol{w})$ of S_i has the following features: 1) it is continuous in $\boldsymbol{w}$, 2) it is concave in w_i, 3) it has a continuous first derivative with respect to w_i, and 4) there exists $r = \{r_1, r_2, ..., r_K\} > 0$ such that $\sum_{i=1}^K r_i \pi_i(\boldsymbol{w})$ is diagonally strictly concave. Following the same methods as in [12, Chapter 5], we can claim that, for this CR networks, the NE is unique and corresponds to the global utility maximization point [11]. In other words, if all SUs use Algorithm 1 to adjust their transmit powers, the payoff of the network will converge to the global utility maximization point, defined as $\max_{\boldsymbol{w}} \frac{1}{K} \sum_{i=1}^K \pi_i(\boldsymbol{w})$, under the power constraint defined in (1).

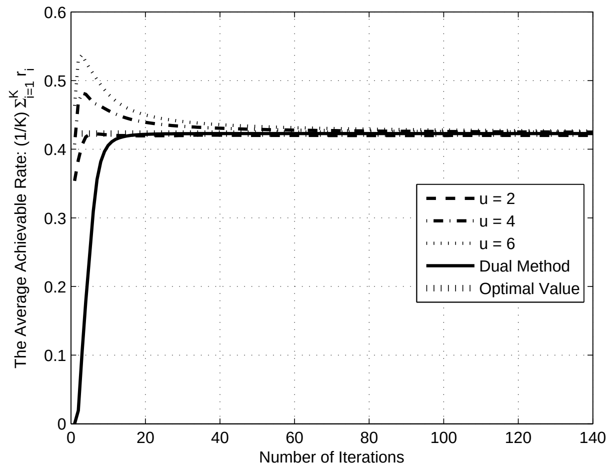

Fig. 2. Comparison of different algorithms, where $M = 1, K = 10, q_k = 2, u_i = \frac{1}{\|g_{i[t]}\|_2^2}$, for $i \in \{1, 2, ..., K\}, k \in \{1, 2, ..., M\}$.

To demonstrate the convergence performance of Algorithm 1, results for the dual methods discussed in [11] are also provided. In this method, S_i iteratively chooses the optimal Lagrange coefficients to maximize its payoff (see [11] for the detailed description of the dual method). In Figure 2, we compare the convergence rates of Algorithm 1 and the dual method. It is observed that, in general, the dual method converges to the optimal value at a faster rate than Algorithm 1. However, each SU in the dual method needs to know the payoff functions of all SUs and is able to search for the optimal power to minimize the objective function in each iteration which incurs much more computational complexity and communication overhead than our proposed algorithm. In addition, as is observed in Figure 2, if the step size are chosen optimally, the convergence rate of Algorithm 1 could outperform that of the dual methods.

Let us consider a more general case as follows. Define the revenue of S_i as $r_i = \alpha_i \log(1 + SNR_i)$ where SNR_i is given in (3) and α_i is a constant. The pricing function charged by all PUs from S_i is given by $c_i = \beta_i \sum_{k=1}^M h_{S_i,P_k} w_i$ where β_i is a constant. Note that this problem is not convex and is, in general, impossible to efficiently find the global optimal

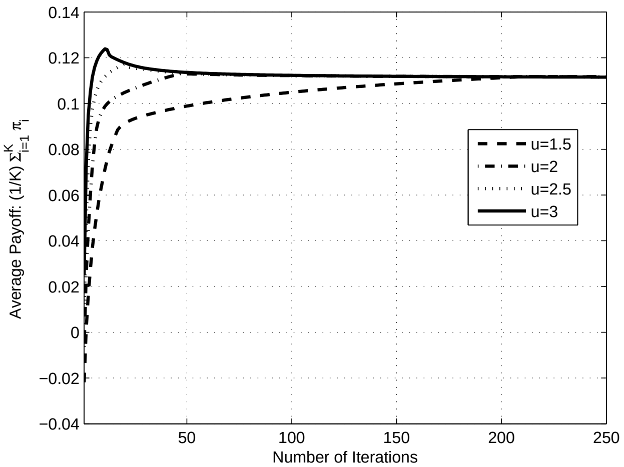

Fig. 3. Convergence rate of Algorithm 1 with different step sizes, where $w_{i[1]}$ is the least squares solution of $\boldsymbol{H}_{PU}\boldsymbol{w}^{\dagger} = \boldsymbol{q}^{\dagger}$, $\alpha_i = 10, \beta_i = 0.1, M = 5, K = 10, q_k = 5, u_{i[t]} = \frac{1}{\|g_{i[t]}\|_2^2}$, for $i \in \{1, 2, ..., K\}, k \in \{1, 2, ..., M\}$.

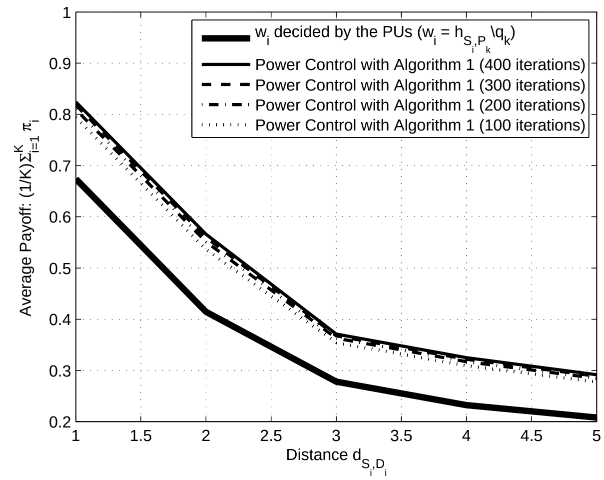

Fig. 5. Payoffs of power control achieved by Algorithm 1 and fixed transmit power ($w_i = h_{S_i,P_k} \backslash q_k$) under different d_{S_i,D_i}, where $\alpha_i = 10, \beta_i = 0.1, M = 5, K = 10, q_k = 2, u_i = \frac{1}{\|g_{i[t]}\|_2^2}$, for $i \in \{1, 2, ..., K\}, k \in \{1, 2, ..., M\}$.

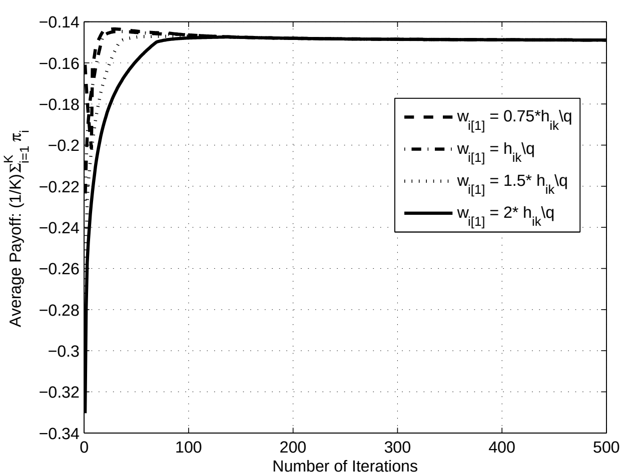

Fig. 4. Convergence rate of Algorithm 1 with different initial values, where $w_{i[1]} = h_{S_i,P_k} \backslash q_k$ means $w_{i[1]}$ is the least squares solution of $\boldsymbol{H}_{PU}\boldsymbol{w}^{\dagger} = \boldsymbol{q}$, $\alpha_i = 10, \beta_i = 0.1, M = 5, K = 10, q_k = 5, u_i = \frac{1}{\|g_{i[t]}\|_2^2}$, for $i \in \{1, 2, ..., K\}, k \in \{1, 2, ..., M\}$.

solution [13]. In this case, Algorithm 1 may not converge to the global utility maximization point, but can still converge to a NE. The convergence results of Algorithm 1 with different parameters in this case is presented in Figures 3 and 4. As observed in Section III, the convergence rate of Algorithm 1 depends on the initial transmit powers of SUs and the iteration step size. It is observed that, if $w_{t[1]}$ has a distance of u_i from a NE, only a few number of iterations are required to approach the performance of this NE. However, if the initial transmit powers are not properly chosen, the adjustment of Algorithm 1 on the transmit powers of SUs may require a large number of iterations. Similarly, an optimal value of u_i should be close to the distance between $w_{i[1]}$ and w_i^*. However, if no information about initial transmit power and the optimal power are known, a large step size may lead to high fluctuations during the first few iterations and a small step size will cause slow convergence rate. The above observation shows the importance of choosing proper combination of initial powers and step sizes. In a practical system, SUs can first send the training code to let PUs to decide proper initial values of SUs' transmit powers. Also, the step size of each iteration, chosen by each SD pair, can be adjusted if more information can be observed or more feedback messages can be received during the communication, i.e., the dual methods. Finding the distributed algorithms to allow iterative improvement of the step size in Algorithm 1 will be our future work.

Let us consider the numerical results of CR networks with Algorithm 1. Denote the distance between two users p and q as $d_{p,q}$ for $p, q \in [S_1, S_2, ..., S_K] \cup [D_1, D_2, ..., D_K] \cup [P_1, P_2, ..., P_M]$ and consider the following channel models, $h_{ij} = \tilde{h}_{ij}/d^{\eta}_{S_i,D_j}$ and $h_{ik} = \tilde{h}_{ik}/d^{\eta}_{S_i,P_k}$, where $\tilde{h}_{ij}$ and $\tilde{h}_{ik}$ are average channel fading coefficients unrelated to the distance of the transmission and η is the channel attenuation exponent. Assume that SUs are located on a regular planar network, i.e., $d_{S_i,P_k} = d_{S_j,P_l}$, $d_{S_i,D_i} = d_{S_j,D_j}$, $d_{S_i,D_j} = \sqrt{d^2_{S_i,S_j} + d^2_{S_j,D_j}}$ and $d_{S_i,S_{i+1}} = d_{D_j,D_{j+1}}$, for $i, j \in \{1, 2, ..., K\}$ and $k, l \in \{1, 2, ..., M\}$. In Figure 5, the average payoff of the network obtained by using Algorithm 1 with different d_{S_i,D_i} is shown and compared with that of the fixed transmit power method. It is observed that the proposed algorithm greatly improves the performance of CR networks. More importantly, even with the small number of iterations, Algorithm 1 can achieve a significant payoff improvement.

V. Conclusion

In this paper, the power control problem for SSS based CR networks with multiple secondary SD pairs and PUs is investigated. A simple distributed algorithm has been proposed to iteratively improve the performance of CR networks. It is proved that the proposed algorithm converges to an ϵ-NE at a speed of $T_\epsilon = O\left(\exp\left(\frac{1}{\epsilon}\right)\right)$. A sub-optimal algorithm is also proposed to converge to a neighborhood of a NE at a rate of $\frac{T_{\epsilon'}}{\log T_{\epsilon'}} = O\left(\frac{1}{\epsilon'}\right)$. Numerical results have been presented to show the convergence rates under different settings.

Appendix A
Proof of Theorem 1

From Definition 1, it is observed that for a NE, a neighborhood can always be found in which all the elements in this neighborhood has lower payoff than the NE. By assuming all payoff functions of SUs to be continuous, the neighborhood of the NE can be regarded as a quasiconcave hull, i.e., for $\boldsymbol{\chi} = [\chi_1, \chi_2, ..., \chi_K]$ neighborhood of a NE, we have $\pi_i(\boldsymbol{w}^* \pm \Delta \boldsymbol{w}) < \pi_i(\boldsymbol{w})$ where $\boldsymbol{w} \in [\boldsymbol{w}^* - \Delta\boldsymbol{w}, \boldsymbol{w}^* + \Delta\boldsymbol{w}]$ and $\Delta w_i \leq \chi_i$. Since Algorithm 1 uses a decreasing step size for iteration, it will finally fall into a quasiconcave hull of the NE. Therefore, if Algorithm 1 is proved to converge to a stationary point in which all elements in a neighborhood of that point has lower payoffs, we can claim that this point is the NE and Algorithm 1 converges. Defining $\boldsymbol{g}_{[t]} = [g_{1[t]}, g_{2[t]}, ..., g_{K[t]}]^\dagger$, we can re-write (9) in Algorithm 1 to the vector form in (11) at the top of next page.

where $\boldsymbol{H}_{PQ}$ is a sub-matrix of $\boldsymbol{H}_{PU}$ which only contains the channel gains connected with one PU observing high interference noises in its rows, and $\boldsymbol{\delta}_{[t]} = \mathrm{diag}[\delta_{1[t]}, \delta_{2[t]}, ..., \delta_{K[t]}]$.

Let us first consider the case that the transmit powers of the SUs always satisfy (1). Denoting a NE achieving power control schemes for all the SUs as $\boldsymbol{w}^* = [w_1^*, w_2^*, ..., w_K^*]^\dagger$, we have

$$\begin{aligned}\frac{1}{T}\sum_{t=1}^{T}\|\boldsymbol{w}_{[t+1]} - \boldsymbol{w}^*\|_2^2 &\overset{(a)}{=} \frac{1}{T}\sum_{t=1}^{T}\|\boldsymbol{w}_{[t]} - \boldsymbol{w}^* - \boldsymbol{\delta}_{[t]}\boldsymbol{g}_{[t]}\|_2^2 \\ &= \frac{1}{T}\sum_{t=1}^{T}\Big[\|\boldsymbol{w}_{[t]} - \boldsymbol{w}^*\|_2^2 - 2\boldsymbol{\delta}_{[t]}\boldsymbol{g}_{[t]}\left(\boldsymbol{w}_{[t]} - \boldsymbol{w}^*\right) \\ &\qquad + \boldsymbol{\delta}_{[t]}^2\|\boldsymbol{g}_{[t]}\|_2^2\Big] \\ &\overset{(b)}{\leq} \frac{1}{T}\sum_{t=1}^{T}\Big[\|\boldsymbol{w}_{[t]} - \boldsymbol{w}^*\|_2^2 - 2\boldsymbol{\delta}_{[t]}\left(\boldsymbol{\pi}\left(\boldsymbol{w}_{[t]}\right) - \boldsymbol{\pi}\left(\boldsymbol{w}^*\right)\right) \\ &\qquad + \boldsymbol{\delta}_{[t]}^2\|\boldsymbol{g}_{[t]}\|_2^2\Big], \end{aligned} \quad (12)$$

where (a) and (b) is obtained by using (5) and (6) in Algorithm 1, respectively. (12) can be rewritten recursively as follows:

$$\begin{aligned}\frac{1}{T}\sum_{t=1}^{T}\left\|\boldsymbol{w}_{[t+1]} - \boldsymbol{w}^*\right\|_2^2 \leq \left\|\boldsymbol{w}_{[1]} - \boldsymbol{w}^*\right\|_2^2 \\ -\frac{2}{T}\sum_{t=1}^{T}\sum_{l=1}^{t}\boldsymbol{\delta}_{[l]}\left(\boldsymbol{\pi}\left(\boldsymbol{w}_{[l]}\right) - \boldsymbol{\pi}\left(\boldsymbol{w}^*\right)\right) \\ +\frac{1}{T}\sum_{t=1}^{T}\sum_{l=1}^{t}\boldsymbol{\delta}_{[l]}^2\left\|\boldsymbol{g}_{[l]}\right\|_2^2. \end{aligned} \quad (13)$$

Note that we have $\frac{1}{T}\sum_{t=1}^{T}\|\boldsymbol{w}_{[t+1]} - \boldsymbol{w}^*\|_2^2 \geq 0$, and the second term in the right-hand-side of (13) can be expressed as follows:

$$\begin{aligned}&\frac{2}{T}\sum_{t=1}^{T}\sum_{l=1}^{t}\boldsymbol{\delta}_{[l]}\left(\boldsymbol{\pi}\left(\boldsymbol{w}_{[l]}\right) - \boldsymbol{\pi}\left(\boldsymbol{w}^*\right)\right) \\ &\geq \frac{2}{T}\sum_{t=1}^{T}\left(\sum_{l=1}^{t}\boldsymbol{\delta}_{[l]}\right)\min_{l\in[1,t]}\left[\boldsymbol{\pi}\left(\boldsymbol{w}_{[l]}\right) - \boldsymbol{\pi}\left(\boldsymbol{w}^*\right)\right] \\ &\overset{(c)}{\geq} \left(\frac{2}{T}\sum_{t=1}^{T}\sum_{l=1}^{t}\boldsymbol{\delta}_{[l]}\right)\frac{1}{T}\sum_{t=1}^{T}\min_{l\in[1,t]}\left[\boldsymbol{\pi}\left(\boldsymbol{w}_{[l]}\right) - \boldsymbol{\pi}\left(\boldsymbol{w}^*\right)\right] \end{aligned} \quad (14)$$

where (c) comes from that fact that $\sum_{l=1}^{t}\frac{1}{l} \geq \frac{1}{T}\sum_{t=1}^{T}\sum_{l=1}^{t}\frac{1}{l}$.

Substituting (14) into (13), we have

$$\begin{aligned}&\frac{1}{T}\sum_{t=1}^{T}\min_{l\in[1,t]}\left[\boldsymbol{\pi}\left(\boldsymbol{w}_{[t]}\right) - \boldsymbol{\pi}\left(\boldsymbol{w}^*\right)\right] \\ &\leq \frac{\left\|\boldsymbol{w}_{[1]} - \boldsymbol{w}^*\right\|_2^2 + \frac{1}{T}\sum\limits_{t=1}^{T}\sum\limits_{l=1}^{t}\boldsymbol{\delta}_{[l]}^2\left\|\boldsymbol{g}_{[l]}\right\|_2^2}{\frac{2}{T}\sum\limits_{t=1}^{T}\sum\limits_{l=1}^{t}\boldsymbol{\delta}_{[l]}}. \end{aligned} \quad (15)$$

Substituting the step size function $\boldsymbol{\delta}_{[l]} = \frac{\boldsymbol{u}}{l}$ for $\boldsymbol{u} = [u_1, u_2, ..., u_K]$ into the above equation and using the result $\sum_{l=1}^{t}\frac{1}{l^2} < \sum_{l=1}^{\infty}\frac{1}{l^2} = \frac{\pi}{6}$, and the properties of Harmonic number: $\sum_{l=1}^{t}\frac{1}{l} = \log t + \frac{1}{2}t^{-1} - \frac{1}{12}t^{-2} + O(t^{-4}) \approx \log t + \frac{1}{2}t^{-1} - \frac{1}{12}t^{-2} + \dot{\kappa}$, the following results can be obtained:

$$\begin{aligned}&\frac{1}{T}\sum_{t=1}^{T}\min_{l\in[1,t]}\left[\boldsymbol{\pi}\left(\boldsymbol{w}_{[l]}\right) - \boldsymbol{\pi}\left(\boldsymbol{w}^*\right)\right] \\ &\overset{(d)}{\leq} \frac{\boldsymbol{w}^+ + \frac{\boldsymbol{w}^+}{T}\sum_{t=1}^{T}\frac{\pi}{6}}{\frac{2}{T}\sum_{t=1}^{T}\left(\log t + 0.5t^{-1} + \dot{\kappa}\right)} \\ &\approx \frac{\ddot{\kappa}}{\frac{2}{T}\sum_{t=1}^{T}\log t + \frac{\log T + 0.5T^{-1} + \dot{\kappa}}{T} + 2\dot{\kappa}} \end{aligned} \quad (16)$$

where (d) is obtained from Assumptions A1) and A2) in Theorem 1. $\ddot{\kappa}$ is a constant defined as $\ddot{\kappa} = \boldsymbol{w}^+ + \boldsymbol{g}^+\frac{\pi}{6}$. Because $\sum_{t=1}^{T}\log t = \Theta(T\log T)$ [2] and hence $\lim_{T\to\infty}\frac{1}{T}\sum_{t=1}^{T}\log t \to \infty$ and $\lim_{T\to\infty}\frac{\log T + 0.5T^{-1}}{T} \to 0$, it can be claimed that Algorithm 1 converges to a stationary point. Assume that for $T \geq T_\epsilon$ and T_ϵ is a very large number, we try to achieve the accuracy of $\frac{1}{T}\sum\limits_{t=1}^{T}\min\limits_{l\in[1,t]}\left[\boldsymbol{\pi}\left(\boldsymbol{w}_{[l]}\right) - \boldsymbol{\pi}\left(\boldsymbol{w}^*\right)\right] = \epsilon$. From (16) and concavity of logarithm function, the following results can be obtained,

$$\begin{aligned}\frac{\ddot{\kappa}}{\epsilon} &\geq \lim_{T_\epsilon\to\infty}\frac{1}{T_\epsilon}\sum_{t=1}^{T_\epsilon}\log t + \frac{\log T_\epsilon + 0.5T_\epsilon^{-1}}{T_\epsilon} + 2\dot{\kappa} \\ &= \lim_{T\to\infty}\log\left(\frac{1}{T_\epsilon}\sum_{t=1}^{T_\epsilon}t\right) = \log\left(\frac{T_\epsilon + 1}{2}\right) \\ &\Rightarrow T_\epsilon = O\left(\exp\left(\frac{1}{\epsilon}\right)\right). \end{aligned}$$

[2]Again, we follow Bachmann-Landau notations: $f = \Theta(g)$ if $f = O(g)$ and $g = O(g)$

$$\boldsymbol{w}_{[t+1]} = \begin{cases} \boldsymbol{w}_{[t]} - \delta_{[t]}\boldsymbol{g}_{[t]}, & \text{if } \boldsymbol{H}_{PU}\boldsymbol{w}_{[t]} \le \boldsymbol{q}_j, \\ \boldsymbol{w}_{[t]} - \delta_{[t]}\boldsymbol{g}_{[t]} - \boldsymbol{H}_{PQ}^{\dagger}\left(\boldsymbol{H}_{PQ}\boldsymbol{H}_{PQ}^{\dagger}\right)^{-1}\left(\boldsymbol{H}_{PQ}\boldsymbol{w}_{[t]} - \boldsymbol{H}_{PQ}\boldsymbol{\delta}_{[t]}\boldsymbol{g}_{[t]} - \boldsymbol{q}\right), & \text{otherwise,} \end{cases} \tag{11}$$

Therefore, we can claim that by using Algorithm 1, the average payoff of SU networks converges to ϵ-NE at a speed of $T_\epsilon = O\left(\exp\left(\frac{1}{\epsilon}\right)\right)$.

Consider the case that at least one PU observes the higher-than-tolerable interference power. By multiplying (11) with $\boldsymbol{H}_{PQ}$, it can be easily shown that $\boldsymbol{H}_{PQ}\boldsymbol{w}_{[t+1]}^{\dagger}$ always converges to $\boldsymbol{q}^{\dagger}$. Substituting $\boldsymbol{H}_{PQ}\boldsymbol{w}_{[t+1]}^{\dagger} = \boldsymbol{q}^{\dagger}$ into (11), (11) can be rewritten as $\boldsymbol{w}_{[t+2]} = \boldsymbol{w}_{[t+1]} - \boldsymbol{\delta}_{[t+1]}\tilde{\boldsymbol{g}}_{[t+1]}\boldsymbol{g}_{[t+1]}$, where $\tilde{\boldsymbol{g}}_{[t+1]} = \boldsymbol{I} + \boldsymbol{H}_{PQ}^{\dagger}\left(\boldsymbol{H}_{PQ}\boldsymbol{H}_{PQ}^{\dagger}\right)^{-1}\boldsymbol{H}_{PQ}$ and $\boldsymbol{I}$ is the identity matrix. If we denote $\boldsymbol{g}_{[t]} = \tilde{\boldsymbol{g}}_{[t]}\boldsymbol{g}_{[t]}$ and assume $\|\tilde{\boldsymbol{g}}_{[t]}\boldsymbol{g}_{[t]}\|_2^2 \le \boldsymbol{g}^{+}$, the same method as the previous case can be used to obtain the similar convergence rate.

Appendix B
Proof of Theorem 2

Following the same steps as in Appendix A, let us consider the case that each SU can only choose from L quantized power levels, i.e., $w_i \in \mathcal{W}_i$. In this case, if the initial point is not appropriately chosen, Algorithm 2 will be unstable for the first few iterations, i.e., fluctuating between different neighborhood of NEs. However, when the number of iterations is large, the transmit powers of SUs will approach to the neighborhood of a NE. Therefore, for a large number of iterations, Algorithm 2 can be regarded as Algorithm 1 with a constant step size $\boldsymbol{\delta}_{[l]} = \hat{\boldsymbol{\delta}}$ where $\hat{\boldsymbol{\delta}} = \frac{\boldsymbol{w}^{+}}{L}$ is a constant. Let us substitute this step size in (15), the following results can be obtained,

$$\begin{aligned} &\frac{1}{T}\sum_{t=1}^{T}\min_{l\in[1,t]}\left[\pi\left(\boldsymbol{w}_{[l]}\right) - \pi\left(\boldsymbol{w}^{*}\right)\right] \\ &\qquad \le \frac{1}{T}\sum_{t=1}^{T}\frac{\boldsymbol{w}^{+} + (\hat{\boldsymbol{\delta}}^2\boldsymbol{g}^{+}/t)\sum_{l=1}^{t} l}{(2\hat{\boldsymbol{\delta}}/t)\sum_{l=1}^{t} l} \\ &\qquad = \frac{1}{T}\sum_{t=1}^{T}\frac{\boldsymbol{w}^{+} + \hat{\boldsymbol{\delta}}^2\boldsymbol{g}^{+}(t+1)}{2\hat{\boldsymbol{\delta}}(t+1)}. \end{aligned} \tag{17}$$

By using the same method as in Appendix A, we can claim that if the step size is fixed, the number of iterations for Algorithm 2 is given by $\frac{\log T'_\epsilon}{T'_\epsilon} = O\left(1/\epsilon'\right)$ where $\epsilon' = \epsilon - \frac{\hat{\boldsymbol{\delta}}\boldsymbol{g}^{+}}{2}$ and $\hat{\boldsymbol{\delta}} = \frac{\boldsymbol{w}^{+}}{L}$.

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